Computing the Resonant Frequency of A Wireless Power Transfer Module by Matching the Inductance and Capacitance Curves

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الملخص:

العديد من التجارب المعملية و التطبيقية أجريت في مجال تحسين نقل الطاقة لاسلكيا و التي اصبحت مهمة لعدة أغراض يومية و طبية و علاجية و غيرها و كان لجانب الحث المغناطيسي لملفات نقل الطاقة نصيبا كبيرا من هذه البحوث و الدراسات و التصاميم و كيفية تحديد تردد الرنين المناسب لتلك الملفات للحصول على أفضل كفاءة لنظام النقل اللاسلكي و هو الموضوع الذي تم اختياره في هذه الورقة ليكون محور بحث و دراسة و استنتاج عبر تطبيق بعض النظريات التي الي تساعد على الوصول الى أفضل النتائج, حيث تم اعتماد الية مطابقة منحنيات المحاثة و السعة لدوائر الرنين المغناطيسي المصممة و ايجاد تردد الرنين لتلك الدوائر.

الكلمات المفتاحية: تردد الرنين, نقل الطاقة الحثي ,طاقة اقتران الرنين.

Abstract:

Many laboratory and applied experiments have been conducted in the field of improving the transmission of energy wirelessly, which has become important for several daily, medical, therapeutic and other purposes. The magnetic induction aspect of the energy transfer files has a large share of these research, studies and designs and how to determine the appropriate resonance frequency for those files to obtain the best efficiency of the wireless transmission system. This is the subject chosen in this paper to be the focus of research, study and conclusion by applying some theories. Where the mechanism is adopted to match the induction and capacity curves of the designed magnetic resonance circuits and to create the resonance frequency of those circuits.

Keywords: Resonant frequency, Inductive energy transfer, Resonant coupling.

I. Introduction:

A great attention paid by manufacturers of portable devices such as phones, laptops and other devices which have become one of the necessities of this era, due to the flexibility of their use and not being restricted to certain places for their use, and out of keenness to find the best solutions to solve the problems they suffer from. These devices are abundant, and one of the most important is their hidden need for energy charging to maintain their optimal performance.

Wireless Power Transfer (WPT) using a near magnetic field area is the subject of much attention recently. At the same time, many studies have researched their different applications, as in many electronic devices. A *WPT* system that is based on magnetically coupled resonance using transmitting T_x and receiving R_x self-resonant coils of a high-quality factor Q was reported in [1] and [2]. Accordingly, a higher coupling coefficient between the T_x and R_x self-resonant coils and the lower losses in each coil are important for effective mid-range *WPT*. In addition, satisfying the optimal impedance-matching conditions according to distance is a key factor for attaining maximum power transfer efficiency in the system. Notably, impedance matching in moving receivers has been found to increase the power transfer efficiency substantially . "Few methods for the impedance matching of a *WPT* system using self-resonant coils of high Q factor have been reported where the mutual inductance between the T_x/ R_x resonant coils and the non-resonant coupling coils was adjusted by mechanically changing the distance between the T_x/ R_x resonant coils and the non-resonant coupling coils" [11]. In an attempt to simulate one of the wireless charging systems for mobile phones and study some of its designs and schemes , it was able to come up with the best applicable design for the resonance circuits within this system by taking into consideration all the results reached by those designers from the operating requirements and all the factors affecting the effectiveness of such a system with operating parameters and their application to our design to calculate the capacitance and inductance of the resulting design for the purpose of calculating and determining the resonant frequency within frequency range of *110 kHz* to *210 kHz* where Figure (1) illustrates the equivalent box diagram of the wireless charging system.

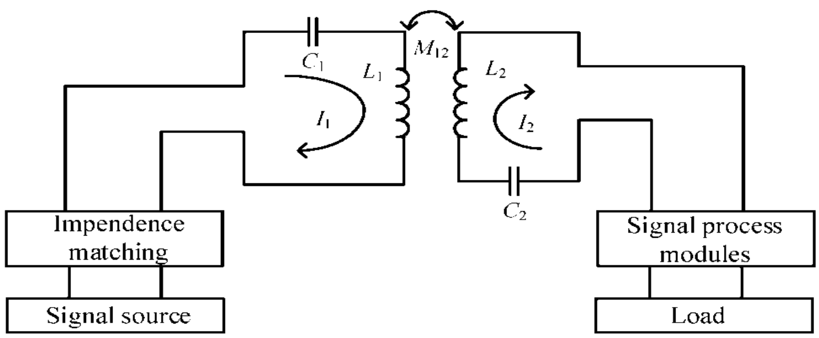


Fig (1): the equivalent box diagram of the wireless charging system

II. Resonance:

One of most interesting characteristic of the series RLC circuit is the phenomenon of resonance ,this phenomena is common among systems that have a tendency to oscillate at a particular frequency. This frequency is called the system's natural frequency in case that this system is driven by a power source at a frequency near to the natural frequency which theoretically produces the amplitude of oscillation to be large.

III. Application of Theory:

The simplest case of *WPT* that can be conveniently described by an impedance approach is provided by two coupled inductors. With reference to Fig.(2) for a matching network of coupled inductances , the *mutual inductance* is denoted with (M):

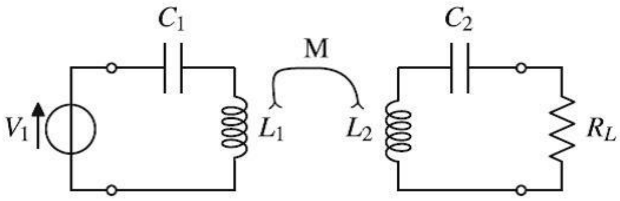


Fig (2): Matching network of coupled inductances.

In a series RLC circuit there becomes a frequency point were the inductive reactance of the inductor becomes equal in value to the capacitive reactance of the capacitor. In other words, $X_L - X_C$. The point at which this occurs is called the Resonant Frequency point, f_r of the circuit, and as analyzing a *series RLC* circuit this resonance, frequency produces a *Series Resonance*.

Firstly, by defining what is already known about series *RLC* circuits:

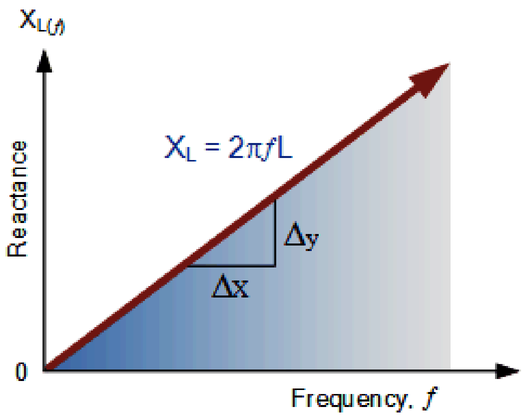
- Inductive Reactance: $X_L = 2\pi fL = \omega L$ (1)
- Capacitive Reactance: $X_C = \frac{1}{2\pi fC} = \frac{1}{\omega C}$ (2)

When $X_L > X_C$ the circuit is inductive, and when $X_C > X_L$ the circuit is Capacitive. The total circuit reactance is $X_T = X_L - X_C$, which means the total circuit impedance is given as:

$Z = \sqrt[2]{{R_0}^2 + {X_T}^2} = R_0 + jX_T$ (3) , where R_0 is the resistance.

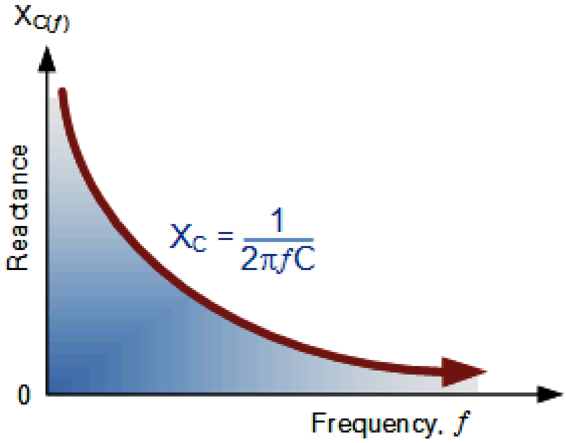
From the above equation for inductive reactance, if either the Frequency or the Inductance is increased the overall inductive reactance value of the inductor would also increase. As the frequency approaches infinity the inductors reactance would also increase towards infinity with the circuit element acting like an open circuit.

However, as the frequency approaches DC, the inductors reactance would decrease to zero, causing the opposite effect acting like a short circuit. This means then that inductive reactance is "Proportional" to frequency as in Fig.(3) and is small at low frequencies and high at higher frequencies and this demonstrated in the following curve:



Fig(3): Inductive Reactance against Frequency.

The inductive reactance value of an inductor increases linearly as the frequency across it increases. Therefore, inductive reactance is positive and is directly proportional to frequency ($X_L \propto f$) in equation (1), and the same is also true for the capacitive reactance formula above in equation (2), but in reverse. If either the frequency or the capacitance is increased the overall capacitive reactance would decrease. "As the frequency approaches infinity the capacitors reactance would reduce to practically zero causing the circuit element to act like a perfect conductor of 0Ω. But as the frequency approaches zero or DC level, the capacitors reactance would rapidly increase up to infinity causing it to act like a very large resistance, becoming more like an open circuit condition" [8] . This means then that capacitive reactance is "Inversely proportional" to frequency for any given value of capacitance and this shown in Fig.(4) below:

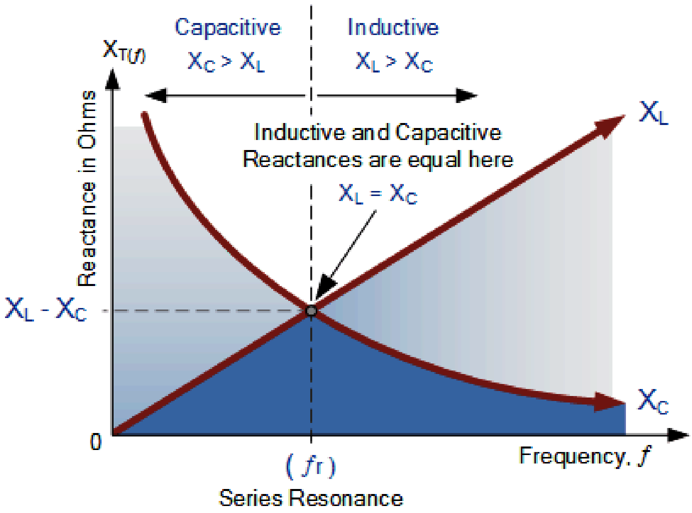


Fig(4): Capacitive Reactance against Frequency.

The graph of capacitive reactance against frequency is a hyperbolic curve, and the reactance value of a capacitor has a very high value at low frequencies but quickly decreases as the frequency across it increases. Therefore, capacitive reactance is negative and is inversely proportional to frequency ($X_C \propto f^{-1}$), therefore it can be seen that the values of these resistances depends upon the frequency of the supply. At a higher frequency X_L is high and at a low frequency X_C is high, then there must be a frequency point were the value of X_L is the same as the value of X_C and there is if the curve for inductive reactance is placed on top of the curve for capacitive reactance so that both curves are on the same axes, the point of intersection will give the *Series Resonance Frequency* point.

III. Series Resonance Frequency:

Electrical resonance occurs in an AC circuit when the effects of the two reactance, which are opposite and equal, cancel each other out as $X_L = X_C$. The point on the following graph in Fig.(5), at which this happens is were the two reactance curves cross each other.



Fig(5): Series Resonance Frequency at f_r.
where: f_r is in Hertz, L is in Henries and C is in Farads.

In a series resonant circuit, the resonant frequency, f_r point can be calculated as follows:

Since; $X_L = X_C$, then $2\pi f L = \frac{1}{2\pi f C}$ (4)

$f^2 = \frac{1}{4\pi^2 LC}$ (5)

$f = \frac{1}{\sqrt[2]{4\pi^2 LC}}$ (6)

This introduces ; $f_r = \frac{1}{2\pi\sqrt[2]{LC}}$ (Hz) Or $\omega_r = \frac{1}{\sqrt[2]{LC}}$ (Rad) (7)

IV. Implementation of Impedance Matching:

This paper aims to introduce an implementation process of frequency range in about 110 kHz to 210 kHz modeling a *WPT* system used in mobile charger . The various frequency ranges have been adopted in different regions based on the frequency allocation for reducing the high-frequency loss and the emission of the electromagnetic field. A *WPT* system designed is mainly composed of the low frequency source, and T_x and R_x coils, and other unit, as shown in Table.(1). In the practical *WPT* device, the low frequency source is generated by a switching device such as a half or full-bridge inverter, and the load unit has a rectifying device to obtain *DC* power from the transferred low frequency source. For accurate analysis of the switching process in the *DC/AC* or *AC/DC* circuit, specialized simulation tools in Matlab environment can be used in a future work. The inductive coupling behavior in the transferring part can be simulated in a numerical computation by the software. Furthermore, the self and mutual inductance of the *WPT* coils can be extracted from the results of the *S-parameter*, and the transferred power in different load and air-gap are predictable. Accordingly, this work provides precise analysis for the formation and distribution of magnetic coupling between the coils. Also, it provides the electrical parameters of the simplified WPT circuit in the wire ports, as shown in Fig.(1). “When the voltage source V_1 with resistance R_0 excites the two-port networks in the T_x, the impedance matrix is expressed in (8), and the self and mutual impedance of coils are determined through (9) and (10):

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{bmatrix} R_0 + j\omega L_1 & j\omega M \\ j\omega M & R_L + j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} R_0 + Z_{11} & Z_{12} \\ Z_{21} & R_L + Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (8)$$

$$Z_{11} = Z_{22} = \frac{(R_0 + S_{11}R_0)(1 - S_{22}) + S_{12}S_{21}R_0}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}} \quad (9)$$

$$Z_{12} = Z_{21} = \frac{2S_{21}(R_0 R_L)}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}} \quad (10)$$

The planar spiral coil which is desired to be simulated is shown in Fig.(6), and the properties of the practical coil are presented in Table(1). The loss caused by the skin effect at low-frequency range was ignored in this study. However, the actual coil was built with 1,650 stranded filaments *Litz*-wire to secure the versatility for the higher frequency systems. Besides, the *WPT* system at low frequency can be free from the skin effect, furthermore, the system needs more turns of coils to produce enough magnetic field, and the transferring distance can be decreased at the low frequency because of the low value of the quality factor; for example, the simulated WPT system at the utility frequency of 110 kHz was introduced and the application implemented with the coil of 45 turns.

The simulations to obtain the *S*-parameters were conducted in the different transfer distance (10, 15, 20, 25, and 30 mm) over the frequency range from 110 kHz to 210 kHz as shown in Fig(7) and (8). The source R_0 and load resistance R_L are set 50 Ω, respectively, during the simulations. The value of self-inductance L_1 and L_2 is constant regardless of the air-gap, and the mutual inductance M and the coupling coefficient k_{12} are correctly calculated based on (11) and (12). The coils in the simulation were constructed in electrical properties similar to a real copper wire.

It was also found that the inductance values present repetitively in the various frequency range.

$$L_1 = \frac{|Z_{11}|}{\omega}\ ,\ L_2 = \frac{|Z_{22}|}{\omega} \qquad (11)$$

$$M = \frac{|Z_{12}|}{\omega}\ ,\quad k_{12} = \frac{M}{\sqrt[2]{L_1 L_2}} \qquad (12)$$

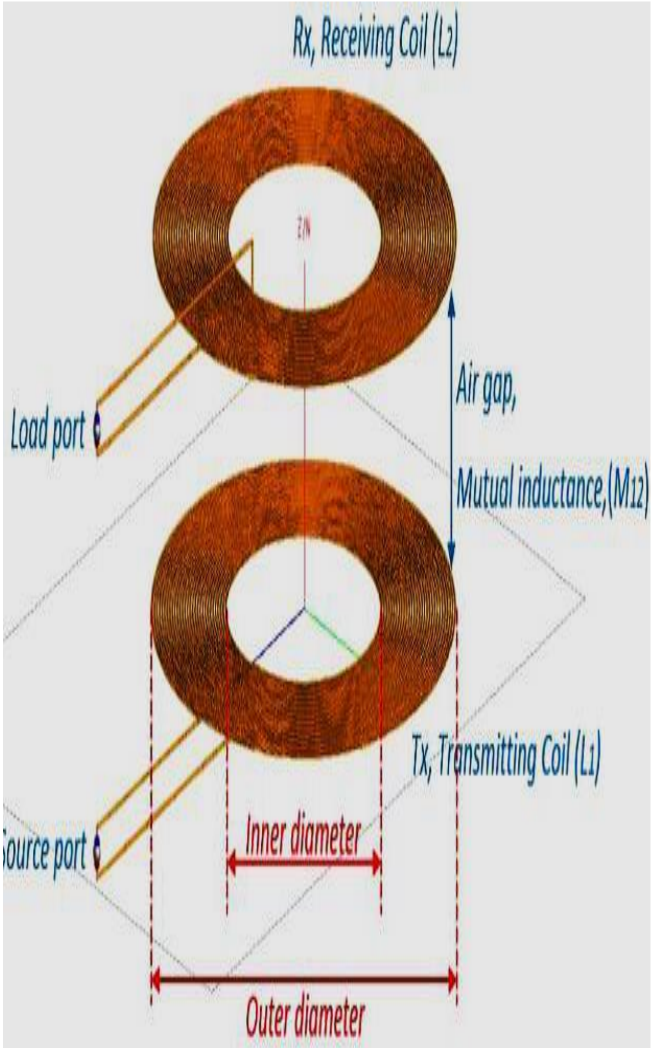


Fig (6): Resonant Coupling Coils.

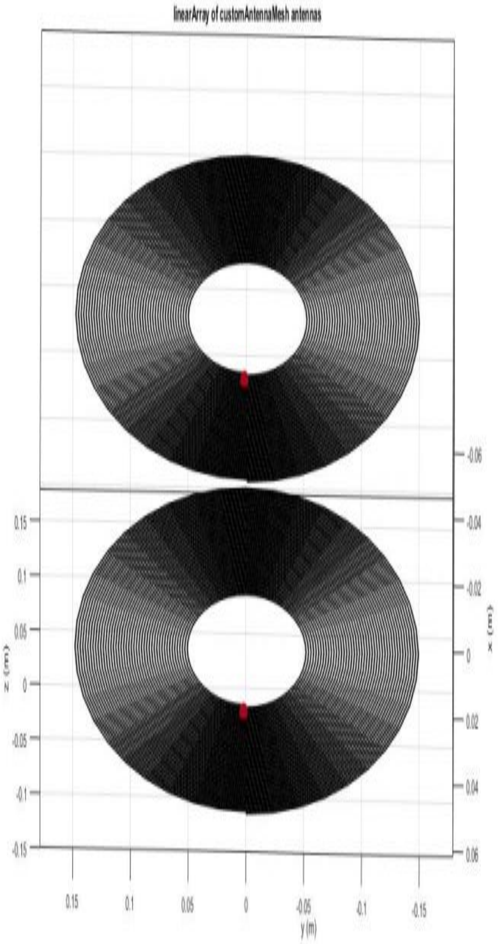


Fig (7): Simulated Resonant Coupling Coils.

The coupling between two spirals increases with decreasing distance between two resonators. This trend is approximately proportional to $\frac{1}{d^3}$, where d is distance between the coupled spirals. Therefore, the system efficiency increases with shorter transfer distance till it reaches the critical coupled regime.

Table (1): Property of the practical coils to be simulated

Property	**Value**
Inner Diameter of *Tx* and *Rx*	**20 mm**
Outer Diameter of *Tx* and *Rx*	**40 mm**
Number of Turns	**45**
Radius of Wire	**0.4 mm**
Medium of Space	**air**

V. The Resulted Matched Curves:

Since Matlab provides a range of numerical computation methods for analyzing data, developing algorithms, and creating models, after performing the basic operations, and applying the most important parameters of the modeled wireless power transfer system to produce all the resulting data that express the impedance of this system, with the frequency response values for both capacitance and inductance of this system, it can be said that it is possible to come up with values of instantaneous curves that express the impedance matching according to the operating frequency and plotting those relationships to display the results of matching curves in the following figures:

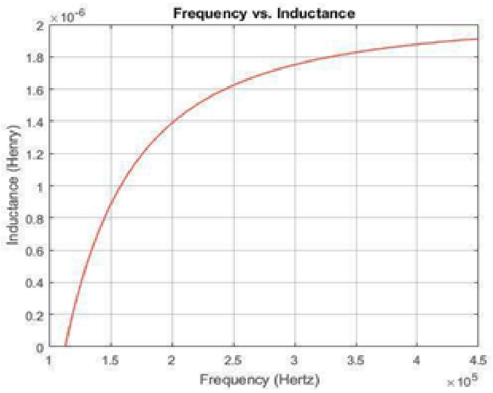


Fig (9): Inductance Vs. Frequency Curve

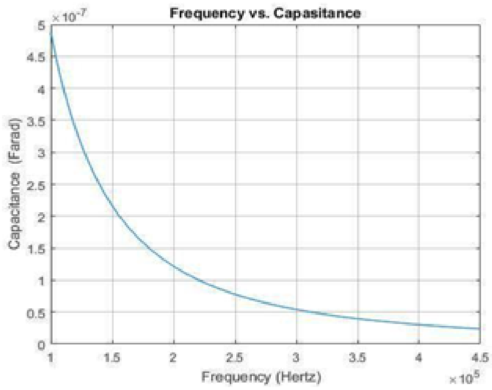
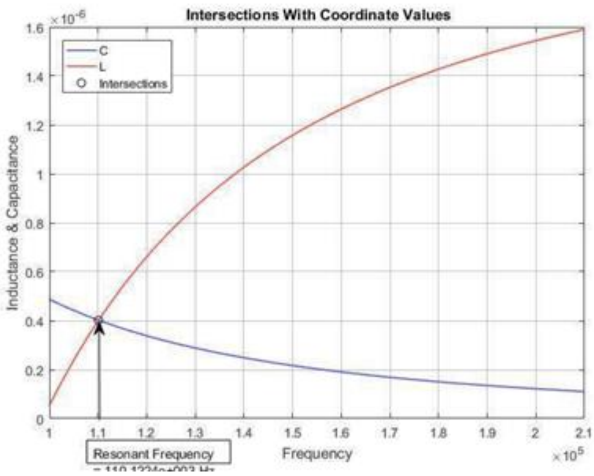


Fig (10): Capacitance Vs. frequency Curve.

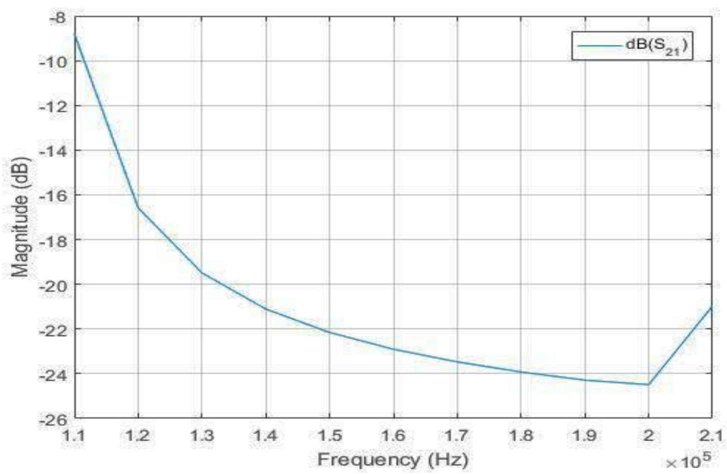


Fig(11): Calculated Resonant Frequency Point

This matching result is mathematically true, when both capacity and inductive reactance equals each other at a value of $(4.017 * 10^{-7})$ to come out with a resonant frequency value of $(110.1224\ kHz)$.

The results from the computational calculation in Table (1) are comparable to the experimental value, and the percentage difference of the self-inductance and the coupling coefficient is under 1.33% and 4.53%, respectively to actual measurements.

One way to evaluate the efficiency of the system is by studying the S_{21} parameter. As presented in Fig.(12), the system efficiency changes rapidly with operating frequency and the coupling strength between the transmitter and receiver resonator at a fixed 20 mm distance. Peak efficiency occurs when the system is operating at its resonant frequency, and the two resonators are strongly coupled.



Fig(12): S_{21} parameter Vs. Operating frequency at 20 mm spacing.

VI. Results Discussion:

The performance of the simulated inductively-coupled *WPT* system is sensitive to the structure of the T_x and R_x coil, and the variation of the air gap (spacing between coils). In this work, the characteristic of the electromagnetic field and the electrical parameters of the *WPT* system were correctly identified through the computational analysis of practical experiment. To demonstrate the *WPT* system, the T_x and R_x coil in the radius of 20 mm were implemented in a number of turns *N* equals 45.25 turns which results S-parameter matrix during the self-mutual inductance of the coils Then, the characteristic of magnetic coupling between the two coils in the series *RLC* circuits combinations in the WPT system at the resonance frequency of 110.1224 kHz was observed by the near-field analysis.

VII. Conclusion:

Through this work, we were able to simulate the process of transmitting power wirelessly through flat spiral coils similar to what was designed in wireless charging devices for mobile phones, by studying them and knowing the most important parameters affecting the charging process. All the physical factors that cannot be applied in such simulations, we took them into account by studying them to know the extent of their impact on the transmission process and its efficiency, as one of the most important of these factors was the magnetic effect of devices close to the real actual system, beside climatic factors in terms of operating temperatures, without ignoring the absence of regressive reflectors used to direct the magnetic flux resulting from the circuits for each of the transmitter and receiver, and our focus in this paper was on determining the appropriate resonant frequency for this design, which was determined to be within the operating frequency range, which we reached after several attempts that were finally concluded with success Where the application error rate did not exceed 5%.

VII. References:

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